MATHEMATICS

RÉSUMÉ OF MATHEMATICS

1. STANDARD OF THE PAPER

The Chief Examiners for Mathematics (Core) 2 and Mathematics (Elective) 2 reported that the standard of the papers compared with that of the pervious years.

2. PERFORMANCE OF CANDIDATES

The Chief Examiner for Mathematics (Core) 2 stated that there was an improvement in performance over that of last year's. The Chief Examiner for Mathematics (Elective) 2 performance of candidates was slightly higher than that of last year

3. SUMMARY OF CANDIDATES' STRENGTHS

(1) The Chief Examiner for Mathematics (Core) 2 stated that Candidates were able to:
- (a) apply Pythagoras theory in solving problems.
- (b) simplify and factorize algebraic expressions.
- (c) to draw trigonometric graphs and use it to sole relevant problems.
- (d) construct cumulative frequency tables and draw graphs of same distribution.
- (e) find the gradient of a line from a given equation.

(2) The Chief Examiner for Mathematics (Elective) 2 outlined that candidates exhibited an improvement in:
- (a) expressing a function as partial fractions.
- (b) finding the Spearman's rank correlation coefficient.
- (c) applying the Quotient rule to differentiate an algebraic fraction.
- (d) finding the magnitude of a resultant force with given magnitudes and directions.
- (e) finding identity element of a given binary operation and the inverse of the given elements.
- (f) using the general formula to find the equation of a circle which passes through three given points.

4. SUMMARY OF CANDIDATES WEAKNESSES

(1) The Chief Examiner for Mathematics (Core) 2 stated that candidates were unable to:
- (a) show evidence of reading values from graphs;
- (b) translate word problems into mathematical equations;
- (c) solve problems on mensuration, geometry and cyclic quadrilaterals.

(2) The Chief Examiner for mathematics (Elective) 2 mentioned that candidates exhibited lack of understanding in:
- (a) applying probability concepts to solve problems.
- (b) finding angles and tensions of an inextensible string fixed at two points.

5. SUGGESTED REMEDIES

(1) The Chief Examiner for Mathematics (Core) 2 the following suggestions to be followed:

(a) In teaching, emphasis should be placed on showing evidence of reading from graphs.

(b) Algebraic concepts should be explained meticulously to help candidates translate word-problems into mathematical equations (statements).

(c) Teahcers should encourage group work among candidates using geometrical figures to enable them solve questions on mensuration and geometry.

(2) The Chief Examiner for Mathematics (Elective) 2 recommended the following to help candidates overcome their weaknesses;

(a) The candidates should be exposed to many exercises on probability.

(b) Teachers should give more attention to the concept of forces relating to tensions in an inextensible string.

MATHEMATICS (CORE) 2

1. STANDARD OF THE PAPER

The standard of this year's paper compared favourably with that of the previous year.

The performance of candidates was better compared to the previous year.

2. SUMMARY OF CANDIDATES' STRENGTHS

The Chief Examiner for Mathematics (Core) listed the strengths of candidates as follows; ability to:

(1) compute probabilities of given events;
(2) construct a cumulative frequency table and draw graph of same distribution;
(3) complete table of values of trigonometric relation in a given interval and draw graph of same;
(4) determine the gradient of a given straight line and finding the equation of a straight line given the gradient and co-ordinates of a point through which the line passes;
(5) draw a Venn diagram for a given information.

3. SUMMARY OF CANDIDATES' WEAKNESSES

The Chief Examiner for Mathematics (Core) listed some of the weaknesses of candidates as follows; ability to:

(1) translate word-problems into mathematical statements;
(2) solve problems in circle theorems;
(3) solve problems involving angles of elevation and depression;
(4) solve problems involving ratio and proportions;
(5) show evidence of reading from a graph.

4. SUGGESTED REMEDIES

(1) Teachers should teach students on how to translate word problems into mathematics statements.
(2) The concept of circle theorems must be explained well in schools.
(3) Teachers should stress on the need for candidates to read and understand the demands of the questions they attempt.
(4) Candidates must be taught to show evidence of reading values on graphs.

5. DETAILED COMMENTS

QUESTION 1

(a) **In a small town, 68% of the people owned Television, 72% owned Radio and 12% owned neither Television nor Radio.**

- **(i) Represent the information on a Venn diagram.**
- **(ii) What percentage of the population owned Television only?**

(b) Boadu and Ansah formed a company and agreed that their annual profit will be shared in the ratio 4:5 respectively. If at the end of the year, Ansah received GH¢5,000.00 more than Boadu, how much was Boadu's share?

Part (a) of this question required the representation of the given information on a Venn diagram. Most candidates answered it correctly. Others on the other hand, omitted essential requirements either for lack of knowledge or otherwise. Instead of $n(U) = 100$, $n(R) = 78$ and $n(T) = 72$ candidates simply stated $U = 100$, $R = 78$ and $T = 72$.

Part (b) required that candidates shared a given sum in the stated ratio, 4:5. Most of the candidates were unable to form the relevant equation to enable them answer the question. A few of them were able to form the required equaton. Some candidates omitted the essential units in the final answer.

Candidates were required to solve part (b) as follows:

$\frac{5}{9}x - \frac{4}{9}x = 5000$

Boadu = $\frac{4}{9}$ x 45,000

Boadu's share = GH¢20,000.00

QUESTION 2

(a) **Make y the subject of the relation: $p = 2x\sqrt{\frac{q\left(1+\frac{r^2}{y^2}\right)}{s}}$**

(b) Given that $m = 3$, $n = -2$ and $x = -1$, evaluate $\frac{2mn^2x}{3m-n}$

In making y the subject of the relation given in 2(a), candidates were required to remove the radical sign by squaring both sides of the relation. Some of the candidates failed to square the factor '$2x$'. This resulted in error in their presentation.

The solution is as follows:

$P^2 = 4x^2\left(\frac{qy^2 + qr^2}{y^2 s}\right)$

$P^2y^2s = 4x^2qy^2 + 4x^2qr^2$

$y^2(p^2s - 4x^2q) = 4x^2r^2q$

$y^2 = \frac{4x^2\ r^2q}{p^2s - 4x^2q}$

$y = \pm\sqrt{\frac{4x^2r^2q}{sp^2 - 4x^2q}}$

Part (b) was quite easily done. The given values were substituted in the expression and same was evaluated. The solution is as follows:

$\frac{2mn^2\ x}{3m-n} = \frac{2(3)(-2^2)\ (-1)}{3(3)-(-2)}$

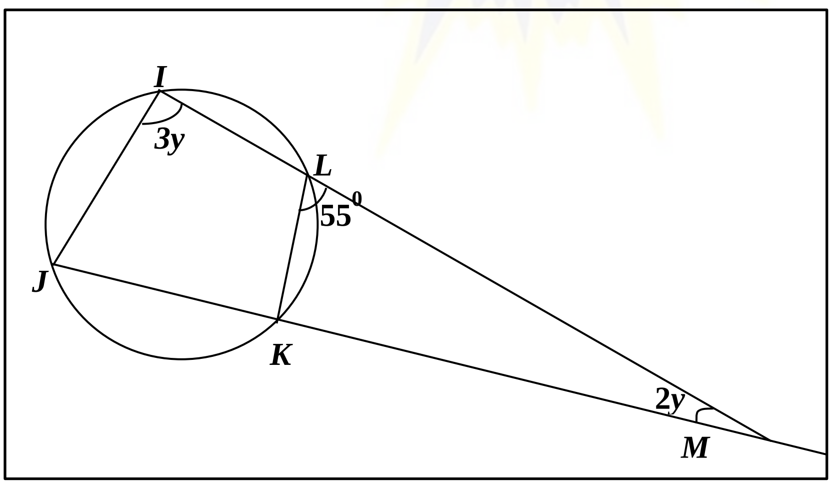
$= \frac{6(4)(-1)}{9+2}$

$= -\frac{24}{11}$

$= -2\ \frac{2}{11}$

<u>QUESTION 3</u>

(a)



NOT DRAWN TO SCALE

In the diagram *IJKL* are points on a circle such that $\angle JIL$= 3*y* and $\angle KML$= 2*y*

If $\angle KLM$= 55^0, find the value of *y*.

(b) Given that tan x = 1, $0^0 \leq x \leq 90^0$, evaluate $\frac{1-sin^2 x}{cos\ x}$

Part (a) was subject to various method aside the solution provided in the marking scheme. Some candidates demonstrated knowledge of circle theorems and geometric facts relating to the triangle. Some of the candidates on the other hand presented solutions that were not judicious.

The solution is as follows:

$\angle LKM + 180 - 3y = 180$

$\angle LKM = 180 - 180 + 3y$

$\angle LKM = 3y$

$55 + 2y + 3y = 180$

$55 + 5y = 180$

$5y = 180 - 55$

$y = 25^0$

Part (b) was quite easily answered. Given tan x = 1, candidates obtained cos $x = \frac{1}{\sqrt{2}}$ and substituted these values in the expression and evaluated same. Other candidates worked out the value of x i.e., tan x = 1, $x = \tan^{-1}(1) = 45^0$ and put 45^0 in place of $-2x$ to evaluate the expression. These attempts were commendable. Candidates were expected to solve it as follows:

$\sin y = \frac{1}{\sqrt{2}}$ $\cos y = \frac{1}{\sqrt{2}}$

$\frac{1-\sin^2 y}{\cos y} = \frac{1-\left(\frac{1}{\sqrt{2}}\right)^2}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$

<u>QUESTION 4</u>

(a) **A cone and a pyramid have equal heights and volumes. If the base area of the pyramid is 154 cm^2, find the radius of the cone. [Take $\pi = \frac{22}{7}$]**

(b) A spherical bowl of radius r cm is a quarter full when 6 litres of water is poured into it. Calculate, correct to three significant figures, the diameter of the bowl. [Take $\pi = \frac{22}{7}$]

In part (a), candidates were able to apply the appropriate formulae to evaluate the radius of the cone.

In part (b), some candidates could not quote the formula of sphere whilst others could not convert 6 liters to cm^3.

Candidates were required to solve (b) as follows:

$\frac{1}{4}$ x $\frac{4}{3}$ x $\frac{22}{7}$ x r^3 = 6000

$r^3 = \frac{6000 \text{ x } 3 \text{ x } 7}{22}$

$= \frac{126000}{22}$

r^3 = 5727.272727

r = 17.8916

Diameter = 2(17.89160)

= 35.8 cm

QUESTION 5

Class	**JHS 1**	**JHS 2**	**JHS 3**
Boys	**32**	**26**	**26**
Girls	**28**	**44**	**36**

The table above shows three classes: JHS 1, JHS 2 and JHS 3 in a school. The three classes were combined to select a prefect. What is the probability that the prefect will be:

(a) **a boy?**

(b) **a girl in JHS 2?**

This question was popular and answered correctly by majority of candidates. Most of the candidates were able to find the total number of students in the school which enabled them to find the probabilities in (a) and (b) correctly.

QUESTION 6

(a) **Copy and complete the table of values for the relation y = 7cos *x* – 3sin *x*.**

x	0^0	30^0	60^0	90^0	120^0	150^0
y	**7.0**			**– 3.0**		

(b) **Using a scale of 2 cm to 30^0 on the x-axis and a scale of 2 cm to 2 units on the *y*-axis, draw the graph of y = 7cos x – 3sin x for $0^0 \leq x \leq 150^0$**

(C) **Use the graph to solve the equations:**

(i) **7cos x = 3sin x;**

(ii) **7cos x = 3.2 + 3sin x.**

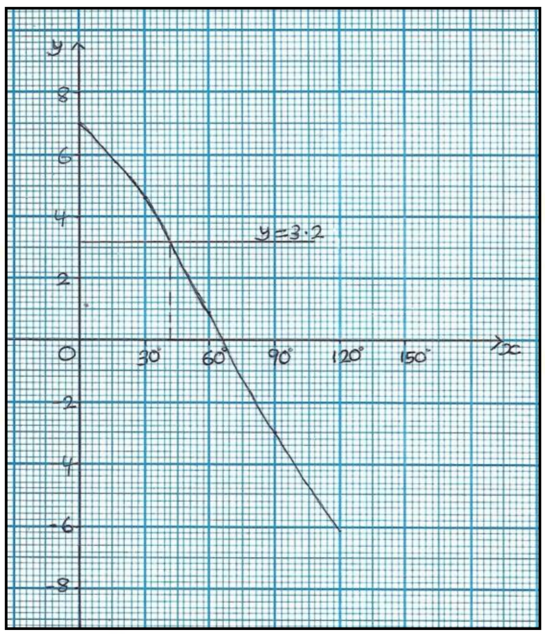
The table of values for the relation was completed correctly. The graph was also plotted. Some candidates however, plotted only the points. They did not join them to obtain a smooth curve. In some cases, the graph drawn by the candidates was like the line of best fit. In part (c)(i) and (c)(ii), candidates transformed the equations to the form $y = 0$ and $y = 3.2$ correctly. Some superimposed these lines on the graph, others did not. Again, some candidates did not use their graph to answer (c)(i) and (c)(ii) as the question demanded.

Candidates were expected to solve the question as follows:

(a)

x	0^0	30^0	60^0	90^0	120^0	150^0
y	7.0	4.9	0.9	-3.0	-6.1	-7.6

(b)



(c)

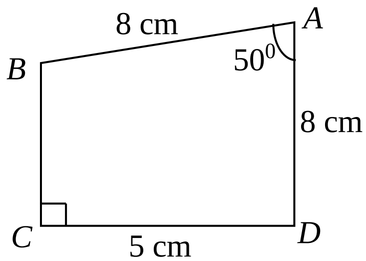
Plotting Graph

From Graph, $y = 0$, $x = 66^0 \pm 3^0$

$y = 3.2$, $x = 42^0 \pm 3^0$

QUESTION 7

(a)



NOT DRAWN TO SCALE

In the diagram $|\overline{AB}| = |\overline{AD}| = 8$ cm and $|\overline{CD}| = 5$ cm. If $\angle BCD = 90^0$ and $\angle BAD = 50^0$, calculate, correct to the nearest whole number:

- **(i)** $|\overline{BD}|$;
- **(ii)** **The area of ΔBCD.**

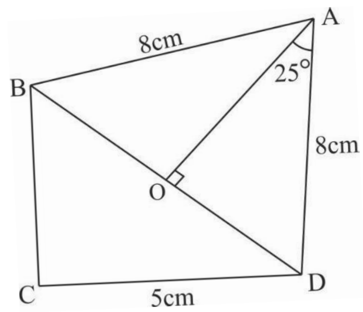
(b) **A man is five times as old as his son. In three years' time, the product of their ages will be 380. Find their present ages.**

In part (a), most candidates assumed that lines *CD* and *AD* were orthogonal. On this premise they proceeded to calculate $|BD|$ and the area of *ABCD* in (i) and (ii) respectively.

This assumption led to wrong answers. The candidates who used either the sine or cosine rules were able to find $|BD|$ with ease, and the area of *ABCD*.

The solution is as follows:

(a)



(i) $|PD|$ = 8 cos 65 = 3.3809

(ii) $|BD|$ = 2 $|PD|$ = 2(3.3809)

= 6.7618

= 7.0 cm

$|BC|$ = $\sqrt{6.7618^2 - 5^2}$

= 4.5521 cm

Area = $\frac{1}{2}$ (5) (4.5521)

= 11.3803

= 11 cm^2

Part (b) was quite easily done by most candidates. From the given information, they obtained the resulting quadratic equation and solved same to get the respective ages of the man and his son.

Candidates were required to solve the question as follows:

Let son's age = x
Father's age = $5x$
In three years, they will be (x+3) and (5x + 3)
(x +3) (5x + 3) = 380
$5x^2 + 18x + 9 = 380$
$5x^2 + 18x - 371 = 0$
$x = 7, \frac{-53}{5}$
$x = 7$
Son's age = 7 years
Father's age = 35 years

QUESTION 8

(a) A market woman purchased a number of plates for GH¢150.00. Four of the plates got broken while transporting them to her shop. By selling the remaining plates at a profit of GH¢1.00 on each, she made a total profit of GH¢6.00. How many plates did she purchase?

(b) If $\frac{1}{32}$, *m*, $\frac{1}{8}$, *n* is in Geometric Progression (G.P), find the values of *m* and *n*.

In part (a), most candidates were able to find the cost price of one plate, profit on each plate as GH¢1.00. Some however could not arrive at the profit equation and hence solve for plates bought.

Candidates were expected to solve the question as follows:

Number of plates = x

Cost of each plate = $\frac{150}{x}$

Cost price = 150 + 6 = 156 cedis

Selling price of each = $\frac{156}{x-4}$

Profit on each is GH¢1.00

$\frac{156}{x-4} - \frac{150}{x} = 1$

$156x - 150(x-4) = x(x-4)$

$x^2 - 10x - 600 = 0$

$(x - 30)(x + 20) = 0$

$x = 30, x = -20$

Plates bought = 30

Part (b) was answered satisfactorily. Given the terms in the GP, candidates obtained the common ratio, r, using the relation $r = \frac{U_2}{U_1} = \frac{U_3}{U_2} = \frac{U_4}{U_3}$ then continued to find the values of the other terms of the sequence.

Candidates were expected to solve the question as follows:

$\frac{m}{\frac{1}{32}} = \frac{\frac{1}{8}}{m}$

$m^2 = \frac{1}{256}$

$m = \frac{1}{16}$

common ratio = $\frac{\frac{1}{16}}{\frac{1}{32}} = 2$

$n = \frac{1}{8} \text{ x } 2 = \frac{1}{4}$

$n = \frac{1}{4}$

QUESTION 9

(a) Two points X and Y, 7 meters apart are on the same horizontal ground. The angles of elevation of a point P from X and Y are 50^0 and 70^0 respectively. Q is a point on XY produced such that $\angle YQP = 90^0$

(i) Illustrate the information in a diagram

(ii) Calculate, correct to two decimal places, the length:

(α) $\overline{XP}$;

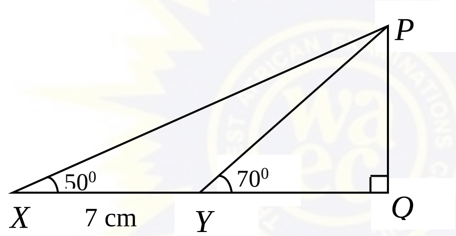
(β) $\overline{YQ}$.

(b) solve the equation: $\frac{3x}{1-x} + \frac{2x}{x+1} = 2$.

In part (a) of the problem, most candidates could not illustrate the information in the appropriate diagram. Q is a point on XY produced such that $\angle YQP = 90^0$ was misunderstood or ignored by some. Instead, Q was located between X and Y, resulting in wrong diagram and subsequently incorrect answers.

Candidates were expected to solve the question as follows:

(a)(i)



(ii) (α) $\angle XYP = 180^0 - 70^0 = 110^0$

$\angle XPY = 180^0 - (110^0 + 50^0) = 20^0$

$\frac{7}{sin20} = \frac{XP}{sin110^0}$

$|XP| = \frac{7\, sin110^0}{sin20^0} = 19.2336$

$= 19.23$ metres

$\angle XPQ = 180^0 - (50^0 + 90^0) = 40^0$

(β) $\frac{XQ}{sin40^0} = \frac{19.23}{sin90^0}$

$|XQ| = \frac{19.23 \text{ X } sin40^0}{sin90^0} = 12.36$

$|YQ| = 12.36\text{m} - 7\text{m} = 5.36\text{m}$

Part (b) was solved with ease, candidates cleared the fractions, expanded the resulting expressions to obtain a quadratic equation which was solved easily.

QUESTION 10

The table shows the age distribution of workers in a company.

Age (years)	**26-30**	**31-35**	**36-40**	**41-45**	**46-50**	**51-55**	**56-60**
Number of Workers	**11**	**24**	**29**	**15**	**10**	**9**	**2**

(a) **Construct a cumulative frequency table and use it to draw a cumulative frequency curve.**

(b) **Use the curve to estimate the:**

(i) **probability of selecting a worker whose age is not more than 45 years:**

(ii) **number of workers who will retire if the retiring age is 50 years and above.**

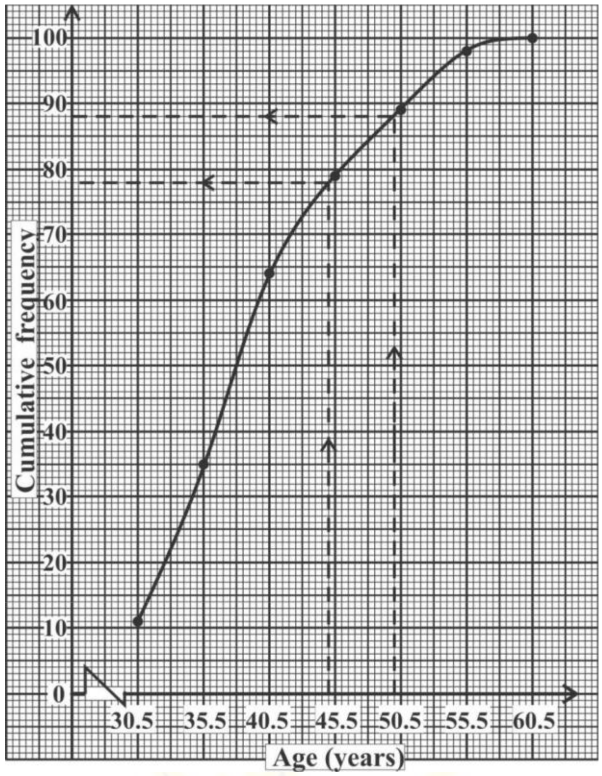
The statistical table for cumulative frequency curve was well done by most candidates. On the graph, most of them used the upper-class boundaries on the horizontal axis and labeled well.

However, most of the candidates could not interpret their graph by showing evidence of reading.

Candidates were required to answer the question as follows:

(a)

Age (year)	Upper class boundaries	Frequency	Commutative frequency
26-30	30.5	11	11
31-35	35.5	24	35
36-40	40.5	29	64
41-45	45.5	15	79
46-50	50.5	10	89
51-55	55.5	9	98
56-60	60.5	2	100



Scale, Axes labelling

(b)(i)

Probability $\frac{77}{100} \pm \frac{1}{100}$ or 0.77 ± 0.01

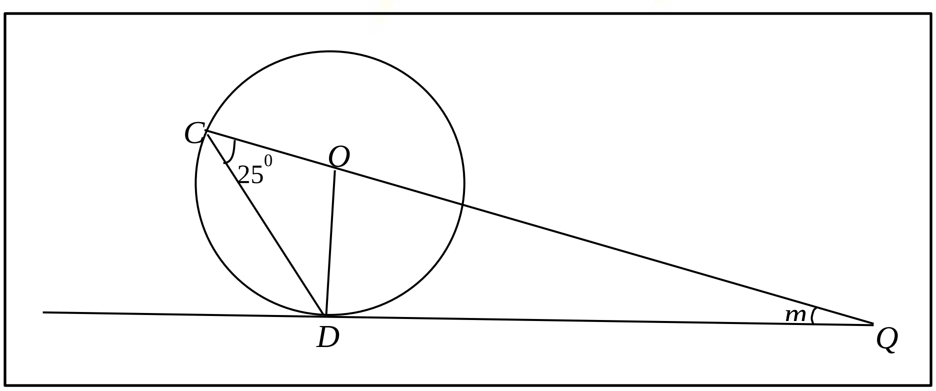
(ii)

88 ± 1

$100 - 88 = 12 \pm 1$ worker

<u>QUESTION 11</u>

(a)



NOT DRAWN TO SCALE

The diagram shows a circle, centre *O*, with *C* and *D* as points on the circumference. $\overline{DQ}$ is a tangent produced at *Q*. Find the value of *m*.

(b) Find the equation of the line which has the same gradient (slope) as $2y + x = 6$ and passes through the point (– 2, 3).

(c) The ratio of the profit, cost of materials and labour in the production of an article is 5: 7: 13 respectively. If the cost of materials is Le 840 more than that of labour, find the total cost of producing the article.

Part (a) of the question was satisfactorily answered by only a few candidates who applied appropriate circle theorem relating an isosceles triangle, to find the value of *m* as indicated in the question.

The solution is as follows:

$\angle DOQ = 50^0$

$\angle CDQ = 115^0$

$m + 115^0 + 25^0 = 180^0$

$m = 180^0 - 140^0 = 40^0$

Part (b) was quite easily solved. Candidates expressed the given equation in the form $y = mx + c$ and deduced the gradient, $m = \frac{-1}{2}$ and later found the equation of the line with gradient $\frac{-1}{2}$ that passed through the point *P* (– 2, 3), using the equation $y - y_, = m(x - x_1)$.

The solution is as follows:

$y = \frac{-1}{2}x + 3$

$m = \frac{-1}{2}$

$y - 3 = \frac{-1}{2}(x + 2)$

$y = \frac{-1}{2}x + 2$

$2y + x - 4 = 0$ or equivalent

Part (c) of this question was not a challenge to most candidates.

Candidates were required to answer the question as follows:

Let x be total cost of producing the article.

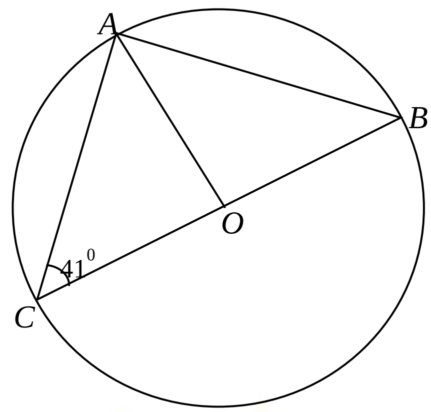
$\frac{13}{25}x - 840 = \frac{7}{25}x$

$13x - 7x = 21000$

$x = 3{,}500$

Total cost of producing the article = Le 3,500.00

<u>QUESTION 12</u>
(a)



NOT DRAWN TO SCALE

In the diagram, O is the centre of the circle ABC and $\angle BCA = 41^0$.
Find:

(i) $\angle BOA$;

(ii) $\angle BAO$.

(b) **The angle of depression of a point, *P*, on the ground from the top, *T*, of a building is 23.6^0. If the horizontal distance from *P* to the base of the building is 50 *m*, calculate, correct to three significant figures, the height of the building.**

(c) **A cow is tied to a post at the centre of a square grazing field of side 25 m by a rope 10 m long. Find, correct to two decimal places the percentage of the field the cow is able to graze on. [Take $\pi = \frac{22}{7}$]**

Part (a) was answered incorrectly, candidates failed to apply the appropriate theorems regarding the angle in a semi circle and angles in an isosceles triangle.

Some candidates assumed that $\angle BOA = \angle AOC = 90^0$, see question. This wrong assumption could not lead to the correct answer. Others who applied the appropriate theorems were able to answer the question with ease.

Part (b) was quite an easy question for those who were able to locate the angle of depression properly. Some candidates could not identify the angle of depression which affected their solution.

Part (c) was quite easily done. Candidates found the area of the grazing field as well as the portion grazeable by the cow and then computed the required percentage.

Candidates were required to solve question 12 (a), (b) and (c) as follows:

(a) $\angle BOA = 2\angle BCA$

(i) $= 2(41) = 82^0$

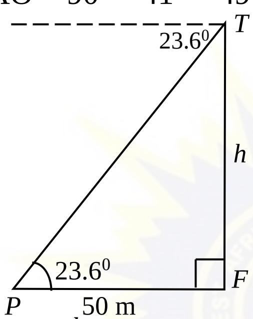
$\angle AOC = 180^0 - 82^0 = 98^0$

$\angle OAC = 180^0 - 41^0 - 98^0$

(ii) $= 41^0$

$\angle BAO = 90^0 - 41^0 = 49^0$

(b)



$\tan 23.6 = \frac{h}{50}$

$h = 50 \tan 23.6 = 21.8469 = 21.8$ m

Area of field = 25 x 25 = 625 m^2

Area of circle the rope makes = $\frac{22}{7}$ x 10^2

= 314.29

(c)

% Grazed = $\frac{314.29}{625}$ x 100%

= 50.29%

<u>QUESTION 13</u>

(a) Given that $f{:}x \rightarrow x + 3$ and $g{:}\ x \longrightarrow x^2$,

(i) **find $g(f(x))$,**

(ii) **evaluate $g(f(2))$.**

(b) **Find what values of x is $\frac{1}{x} + \frac{1}{x+2}$ undefined?**

(c) **Given that $f(x) = \frac{k}{x+1} + \frac{6}{x+2}$ and $f\,(5) = 8$, find the value of k.**

Part (a) of the question was unpopular and challenging to most of those who attempted it. This borders on composite functions, which obviously was unfamiliar to most of the candidates. They could not substitute the required function into the equation.

Part (b) was done satisfactorily. Candidates demonstrated knowledge of the circumstance under which a rational expression may be undefined by equating the denominator to zero to get the required solution.

Part (c) was also answered correctly by most candidates. The relevant substitution was made, and the equation was simplified and solved for the value of the unknown quantity, *k*.

Candidates were required to answer the question 13 (a), (b) and (c) as follows:

(a) (i)

$$G(f(x)) = (x+3)^2$$
$$= x^2 + 6x + 9$$
$$G(f(x)) = x^2 + 6x + 9$$

(ii)

$$G(f(2)) = 2^2 + 6(2) + 9$$
$$= 25$$

(b)

$$x(x+2) = 0$$
$$x = 0$$
$$x = -2$$

(c)

$$8 = \frac{k}{5+1} + \frac{6}{5+2}$$
$$8 = \frac{k}{6} + \frac{6}{7}$$
$$\frac{k}{6} = \frac{50}{7}$$
$$k = 42\frac{6}{7}$$

MATHEMATICS (ELECTIVE) 2

1. **STANDARD OF THE PAPER**

The Chief Examiner for Mathematics (Elective) reported that the standard of the paper compared favourably with that of the previous years.

Candidates' performance was a notch higher than the previous year.

2. **SUMMARY OF CANDIDATES' STRENGTHS**

The Chief Examiner for Mathematics (Elective) listed the strengths of candidates as follows; ability to:

(1) find identity element and inverse of a binary operation;
(2) find composite functions;
(3) resolve forces into components;
(4) find the magnitude of the resultant of various forces;
(5) resolve functions into partial fractions;
(6) solve problems in probability;
(7) identify nature of roots of a quadratic function.

3. **SUMMARY OF CANDIDATES' WEAKNESSES**

The Chief Examiner for Mathematics (Elective) listed some of the weakness of candidates as difficulty in:

(1) drawing a Histogram;
(2) calculating of Spearman's Rank Correlation Coefficient;
(3) solving problems involving both Arithmetic/Geometric Progression;
(4) using properties of the scalar (Dot) Product to solve problems;
(5) drawing accurate tree-body diagrams to represent problems in mathematics;
(6) using of cosine/sine ranks appropriately in a triangle.

4. **SUGGESTED REMEDIES FOR THE WEAKNESSES**

(1) Teachers should give equal attention to all topics in the syllabus rather than specializing on some topics.
(2) Candidates should be given more exercises for them to have good command of the topics in the syllabus.
(3) Candidates should be encouraged to show all steps used in solving a problem clearly without jumping the steps in arriving at the solution.
(4) In general, teachers should pay attention to the weaknesses outlined above.

5. **DETAILED COMMENTS**

QUESTION 1

A binary operation Δ is defined on the set of real numbers, *R*, by *x* Δ *y* = *x* + *y* + 10. Find the:

(a) identity element;

(b) inverses of 3 and – 5 under Δ

Candidates were required to find the identity element for the binary operation $x \, \Delta \, y = x + y + 10$ and to find the inverse of 3 and -5 under the operation. The question was attempted by most candidates and was well handled. Thus, a good number of them were able to find the identity element and the inverse.

Candidates were required to solve (a) and (b) as follows:

(a)
$$\begin{aligned} x\Delta y &= x + y + 10 \\ x\Delta\, e &= x + e + 10 = x \\ x + e + 10 &= x \\ e &= -10 \end{aligned}$$

(b)
$$\begin{aligned} 3\,\Delta\, x^{-1} &= 3 + x^{-1} + 10 = -10 \\ 3 + x^{-1} + 10 &= -10 \Rightarrow\ x^{-1} = -23 \\ -5\Delta\, x^{-1} &= -5 + x^{-1} + 10 = -10 \\ -5 + x^{-1} + 10 &= -10 \Rightarrow\ x^{-1} = -15 \end{aligned}$$

QUESTION 2

Evaluate $\int_2^4 \left(\frac{x^3 + 3}{x^2}\right) dx$.

Candidates were required to evaluate the integral $\int_2^4 \left(\frac{x^3 + 3}{x^2}\right) dx$. The question was attempted by most of the candidates and performance was good. Few candidates could not divide through by x^2 to obtain $x + 3x^{-2}$ before integrating. Such candidates found it difficult to integrate.

Candidates were required to solve question 2 as follows:

$$\begin{aligned} \int_2^4 \frac{x^3 + 3}{x^2}\, dx &= \int_2^4 (x + 3x^{-2})\, dx \\ &= \left[\frac{x^2}{2} + \frac{3x^{-1}}{-1}\right]_2^4 \\ &= \left[\frac{x^2}{2} - \frac{3}{x}\right]_2^4 \end{aligned}$$

$$= \left(8 - \tfrac{3}{4}\right) - \left(2 - \tfrac{3}{2}\right)$$
$$= 7.25 - 0.5$$
$$= 6.75$$

QUESTION 3

(a) **Two functions f and g are defined on the set of real numbers, *R* by *f*: $x \longrightarrow x^2 - 1$ and *g* : $x \rightarrow x+2$. Find $f \circ g(-2)$**

(b) **A bus has 6 seats and there are 8 passengers. In how many ways can the bus be filled?**

The question was in two parts, (a) and (b).

The question was attempted by most of the candidates. The part (a) was answered very well; the second part posed a problem for some of them. That part (b) required the use of permutation instead of combination. Candidates were required to answer the question as follows:

(a)
$$f: x \to x^2 - 1,\ g: x \to x + 2$$
$$f\ o\ g = (x+2)^2 - 1$$
$$= x^2 + 4x + 4 - 1$$
$$= x^2 + 4x + 3$$
$$f\ o\ g(-2) = (-2)^2 + 4(-2) + 3$$
$$= 4 - 8 + 3$$
$$= -1$$

(b)
$$8_{P_6} = \frac{8!}{(8-6)!} = \frac{8!}{2!}$$
$$= 20160 ways$$

QUESTION 4

Express $\frac{1}{x^2 - 16}$ in partial fractions.

A good number of candidates attempted the question and were able to resolve the expression in partial fractions.

Candidates were required to answer the question as follows:

$$\frac{1}{x^2 - 16} = \frac{1}{(x-4)(x+4)}$$

$$\frac{1}{(x-4)(x+4)} \equiv \frac{P}{x-4} + \frac{Q}{x+4}$$

$$1$$

$$= P(x+4) + Q(x-4)$$

$$put\ x = 4$$

$$1 = 8P$$

$$P = \frac{1}{8}$$

$$Next\ put\ x = -4, 1 = -8Q$$

$$\therefore Q = -\frac{1}{8}$$

$$\frac{1}{x^2-16} \equiv \frac{\frac{1}{8}}{x-4} + \frac{-\frac{1}{8}}{x+4}$$

$$\equiv \frac{1}{8(x-4)} - \frac{1}{8(x+4)}$$

QUESTION 5

The table shows the marks scored by some students in a class test.

Marks	**11 – 14**	**15 - 18**	**19 – 22**	**23 - 26**	**27 - 30**	**31 - 34**	**35 - 38**
No. of students	**4**	**5**	**18**	**31**	**25**	**14**	**3**

(a) **Draw a histogram for the distribution.**

(b) **Use the histogram to estimate the modal score, correct to one decimal place.**

Candidates were required to draw a histogram from a frequency table and use the histogram to estimate the modal mark. The question was attempted by most of the candidates. They were able to construct the table but could not draw the histogram correctly by using either the midpoints or the class boundaries. They could not show the exact positions of the class boundaries or the midpoints. Candidates' performance was generally very poor.

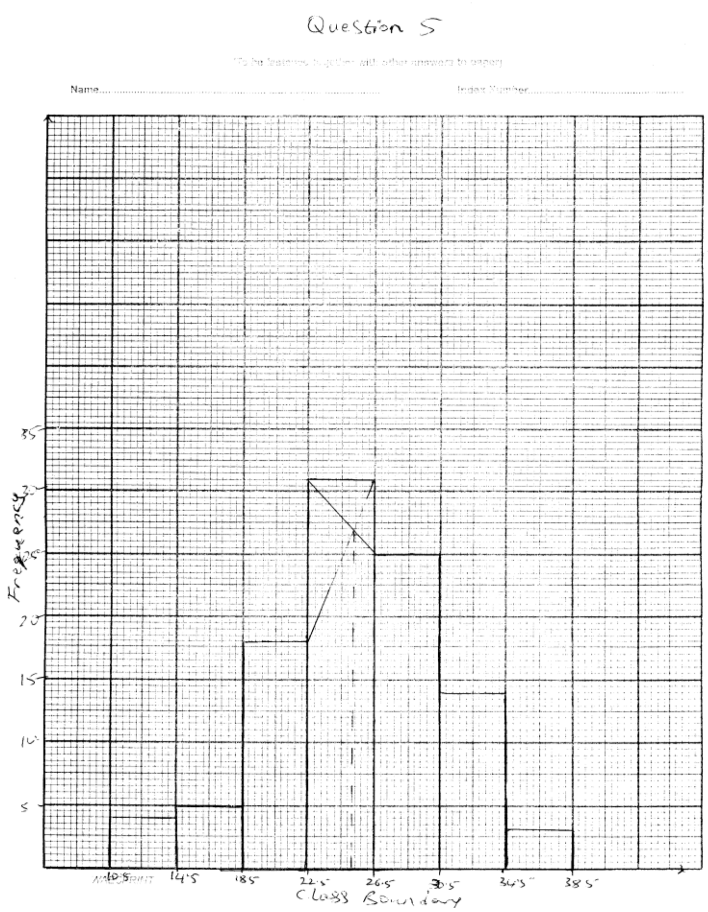
Candidates were required to answer the question as follows:

(a)

Class Boundary	f
10.5 – 14.5	4
14.5 – 18.5	5
18.5 – 22.5	18
22.5 –26.5	31
26.5 – 30.5	25
30.5 – 34.5	14
34.5 – 38.5	3

(b) From the graph

Mode = 25.2 $\pm$ 0.1



<u>QUESTION 6</u>

A bag contains 10 black and 5 yellow identical balls. Two balls are picked at random from the bag one after the other without replacement. Calculate the probability that they are:

(a) both black;

(b) of the same colour.

Candidates were required to find the probability of picking two balls at random from a bag without replacement. Almost all candidates who attempted the question got the answer correct. Most candidates used combination to solve the problem.

Candidates were required to answer the question as follows:

(a) $black = 10, yellow = 5, total = 15$

P(both black)$\frac{\binom{10}{2}\binom{5}{0}}{\binom{15}{2}} = \frac{45\times 1}{105}$

$= \frac{45}{105}$

$= \frac{3}{7}$ or 0.4286

(b) P(the same colour) $= \frac{\binom{10}{2}\binom{5}{0} + \binom{10}{0}\binom{5}{2}}{\binom{15}{2}}$

$= \frac{(45\times 1)+(1\times 10)}{105}$

$= \frac{45+10}{105}$

$= \frac{55}{105}$

$= \frac{11}{21}$ or 0.5238

<u>QUESTION 7</u>

Force F_1 (24 N, 120^0), F_2(18N, 240^0) and F_3(12N,300^0) act at a point. Find, correct to two decimal places, the magnitude of their resultant force.

Candidates were required to find the magnitude of the resultant of three forces acting at a point. A good number of candidates attempted this question and it was well answered by most of them. They were able to resolve the forces into components, simplified and obtained the magnitude of the forces. Few candidates interchanged the *x* and *y* components of the forces which led to loss of marks.

Candidates were required to answer the question as follows:

$$F_R = F_1 + F_2 + F_3$$

$$F_R = \begin{pmatrix} 24sin120^0 \\ 24cos120^0 \end{pmatrix} + \begin{pmatrix} 18sin240^0 \\ 18cos240^0 \end{pmatrix} + \begin{pmatrix} 12sin300^0 \\ 12cos300^0 \end{pmatrix}$$

$$= \begin{pmatrix} 12\sqrt{3} \\ -12 \end{pmatrix} + \begin{pmatrix} -9\sqrt{3} \\ -9 \end{pmatrix} + \begin{pmatrix} -6\sqrt{3} \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} -3\sqrt{3} \\ -15 \end{pmatrix}$$

$$|F_R| = \sqrt{\left(3\sqrt{3}\right)^2 + (15)^2} = \sqrt{27 + 225} = \sqrt{252}$$

$$= 15.87N$$

QUESTION 8

The vectors *p*, *q* and *r* are mutually perpendicular with $|q|$ = 3 and $|r|$ = $\sqrt{5.4}$. If the vectors *X* = 3*p* + 5*q* + 7*r* and *Y* = 2*p* + 3*q* – 5*r* are perpendicular, find $|p|$.

Candidates were required to find one of three mutually perpendicular vectors. In the process they were required to apply the properties of the dot product. Most of the candidates avoided the question. Very few of them who attempted gave answers which showed that they have little knowledge of dot product of (perpendicular) vectors which the question required. The question was poorly answered.

Candidates were required to answer the question as follows:

$\underline{X} = 3p = 5q + 7r$ and $\underline{Y} = 2p + 3q - 5r$

$$\underline{X} \cdot \underline{Y} = \left(3\underline{p} + 5\underline{q} + 7\underline{r}\right) \cdot \left(2\underline{p} + 3\underline{q} - 5\underline{r}\right) = 0$$

$$= 6\underline{p} \cdot \underline{p} + 9\underline{p} \cdot \underline{q} - 15\underline{p} \cdot \underline{r} + 10\underline{q} \cdot \underline{p} + 15\underline{q} \cdot \underline{p}$$

$$-25\underline{q} \cdot \underline{r} + 14\underline{r} \cdot \underline{p} + 21\underline{r} \cdot \underline{q} - 35\underline{r} \cdot \underline{r} = 0$$

$$6|p|^2 + 15|q^2| - 35|r^2| = 0$$

$$6|p^2| = 54$$

$$\therefore |p| = 3$$

QUESTION 9

(a) **If (p + 1) x^2 + 4px + (2p + 3) = 0 has equal roots, find the integral value of p.**

(b) **Solve for x and y in the equations: log (x – 1) + 2 log y = 2 log 3;**
log x + log y = log 6

Most of the candidates attempted this question and scored good marks for the part (a). A good number of the candidates could not remove the logs in the part (b).

Candidates were required to answer the question as follows:

(a) $(p+1)x^2 + 4px + (2p+3) = 0$

For equal roots: $b^2 - 4ac = 0$

$$16p^2 - 4(p+1)(2p+3) = 0$$
$$8p^2 - 20p - 12 = 0$$
$$2p^2 - 5p - 3 = 0$$
$$(p-3)(2p+1) = 0$$
$$p = -\frac{1}{2} \text{ or } p = 3$$
$$\therefore p = 3$$

(b) $\log(x-1) + 2\log y = 2\log 3$

$$\log(x-1) + \log y^2 = \log 3^2$$
$$y^2(x-1) = 9 \text{ ………………..…} (1)$$
$$\log x + \log y = \log 6$$
$$xy = 6 \text{ ………………} (2)$$
$$x = \frac{6}{y}$$
$$y^2\left(\frac{6}{y} - 1\right) = 9$$
$$y^2 - 6y + 9 = 0$$
$$y^2 - 3y - 3y + 9 = 0$$
$$y(y-3) - 3(y-3) = 0$$
$$(y-3)(y-3) = 0$$
$$y = 3$$
$$x = \frac{6}{3}$$
$$\therefore x = 2$$

QUESTION 10

(a) Differentiate $y = \frac{3x}{1+x^2}$ with respect to x.

(b) Find the equation of the circle that passes through (2,3), (4,2) and (1,11).

(a) The part (a) was attempted by most of the candidates. Some candidates could not use the quotient rule correctly.

(b) The part (b) was correctly answered by most of the candidates. Candidates did the correct substitution to generate the three equations but were unable to solve the three equations well.

Candidates were required to answer the question as follows:

(a) $y = \frac{3x}{1+x^2}$

$$\frac{dy}{dx} = \frac{(1+x^2)\cdot 3 - 3x\cdot(2x)}{(1+x^2)^2}.$$

$$= \frac{3+3x^2-6x^2}{(1+x^2)^2}$$

$$= \frac{3-3x^2}{(1+x^2)^2}$$

$$= \frac{3(1-x^2)}{(1+x^2)^2}$$

(b)

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

At (2,3), $4 + 9 + 4g + 6f + c = 0$

$4g + 6f + c = -13$ ……(i)

At (4,2), $16 + 4 + 8g + 4f + c = 0$

$8g + 4f + c = -20$……(ii)

At (1,11), $1 + 121 + 2g + 22f + c = 0$

$2g + 22f + c = -122$.....(iii)

(ii) – (i), $4g + 2f = -7$……..(iv)

(ii) – (iii), $-6g + 18f = -102$….(v)

$g = \frac{-11}{2}$

$f = -15$

$c = 54$

$$x^2 + y^2 + 2\left(-\frac{11}{2}\right)x + 2\left(-\frac{15}{2}\right)y + 6 = 0$$

$$x^2 + y^2 - 11x - 15y + 54 = 0$$

QUESTION 11

When the terms of a Geometric Progression (G.P.) with common ratio r = 2 is added to the corresponding terms of an Arithmetic Progression (A.P.), a new sequence is formed. If the first terms of the G. P. and A. P. are the same and the first three terms of the new sequence are 3, 7 and 11 respectively, find the n^{th} term of the new sequence.

The question was on *AP* and *GP* was answered by most of the candidates. A few of the candidates who attempted it could not make any headway. Candidates were required to answer the question as follows:

$G.P = a, 2a, 4a$

$A.P = a, a+d, a+2d.$

$G.P + A.P = 2a + 3a+d + 5a+2d$

$2a = 3$

$a = \frac{3}{2}$

$3a + d = 7$

$d = \frac{7}{4}$

For new sequence

$a = 3, d = 4$

$a_n = 3 + 4(n-1)$

$= 4n - 1$

QUESTION 12

(a) **The probabilities that Golu, Kofi and Barry will win a competition are $\frac{1}{3}, \frac{2}{5}$ and $\frac{1}{2}$ respectively. Find the probability that only two of them wins the competition**

(b) **Ten eggs are picked successively with replacement from a lot containing 10% defective eggs. Find the probability that at least two are defective.**

Candidates were required to find the probabilities of events in different statement and cases. The part (a) was well answered by most of the candidates.

The part (b) however, which required the binomial probability was poorly answered.

Candidates were required to answer the question as follows:

(a)
$$P(G) = \frac{1}{3}, P(K) = \frac{2}{5}, P(B) = \frac{1}{2}$$
$$P(G^1) = \frac{2}{3}, P(K^1) = \frac{3}{5}, P(B^1) = \frac{1}{2}$$
G wins, K wins, B lose $= \frac{1}{3} \times \frac{2}{5} \times \frac{1}{2} = \frac{1}{15}$

G wins, K lose, B wins $= \frac{1}{3} \times \frac{3}{5} \times \frac{1}{2} = \frac{1}{10}$

G lose, K wins, B wins $= \frac{2}{3} \times \frac{2}{5} \times \frac{1}{2} = \frac{2}{15}$

P(only two wins) $= \frac{1}{15} + \frac{1}{10} + \frac{2}{15} = \frac{9}{30}$

$= \frac{3}{10}$ or 0.3

(b)
$$n = 10, \quad p = \frac{1}{10}, \quad q = \frac{9}{10}$$
$$P(X \geq 2) = 1 - P(X < 2)$$
$$= 1 - [P(X = 0) + P(X = 1)]$$
$$P(X = 0) = \binom{10}{0}\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{10} = 0.3487$$
$$P(X = 1) = \binom{10}{1}\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^9 = 0.3874$$
$$P(X \geq 2) = 1 - (0.3487 + 0.3874))$$
$$= 1 - 0.7361$$
$$\therefore P(X \geq 2) = 0.2639$$

QUESTION 13

The marks awarded by three examiners are given in the table:

Candidate	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
***Examiner* I**	**90**	**88**	**71**	**65**	**32**	**72**	**70**	**41**	**38**	**14**
***Examiner* II**	**89**	**92**	**70**	**68**	**35**	**66**	**72**	**39**	**40**	**16**
***Examiner* III**	**88**	**89**	**71**	**67**	**36**	**70**	**69**	**38**	**39**	**15**

(a) Calculate the Spearman's correlation coefficient of the marks awarded by:
- **(i) Examiners I and II;**
- **(ii) Examiners I and III;**
- **(iii) Examiners II and II.**

(b) Use your results in (a) to determine which of the examiners agree most.

Candidates were required to calculate the spearman's rank correlation coefficient of the marks awarded by some examiners and conclude which examiners agreed most.

(a) Very few of the candidates attempted this question. The performance of the candidates was very poor. Most of the candidates could not do the ranking and squaring to be used in the formula. Most of them could not write the formular correctly and therefore mixed up the calculations.

Those who managed to solve the problem could not use their results to conclude which of the examiners agreed most.

Candidates were required to answer the question as follows:

13.(a)

Student	R_I	R_{II}	R_{III}	$d^2_{I,II}=(R_I - R_{II})^2$	$d^2_{I,\ III} = (R_I - R_{III})^2$	$d^2_{II,III} = (R_{II} - R_{III})^2$
A	1	2	2	1	1	0
B	2	1	1	1	1	0
C	4	4	3	0	1	1
D	6	5	6	1	0	1
E	9	9	9	0	0	0
F	3	6	4	9	1	4
G	5	3	5	4	0	4
H	7	8	8	1	1	0
I	8	7	7	1	1	0
J	10	10	10	0	0	0
				$\sum d^2_{I,\ II} = 18$	$\sum d^2_{I,\ III} = 6$	$\sum d^2_{II,\ III} = 10$

$$r_{12} = 1 - \frac{6\sum di^2}{n(n^2-1)}$$

$$1 - \frac{6 \times 18}{10 \times 99} = \frac{49}{55} = 0.8909$$

$$r_{13} = 1 - \frac{6 \times 6}{10 \times 99} = \frac{53}{55} = 0.9636$$

$$r_{23} = 1 - \frac{6 \times 10}{10 \times 99} = \frac{31}{33} = 0.9394$$

Examiners I and III agree most

(b) Examiners I and III agree most

QUESTION 14

The ends X and Y of an inextensible string 27 m long are fixed at two points on the same horizontal line which are 20 m apart. A particle of mass 7.5 kg is suspended from a point p on the string 12 m from X

(a) Illustrate this information in a diagram.

(b) Calculate, correct to two decimal places, $\angle YXP$ and $\angle XYP$.

(c) Find, correct to the nearest hundredth, the magnitudes of the tensions in the string.

[Take $g = 10\ m\ s^{-2}$]

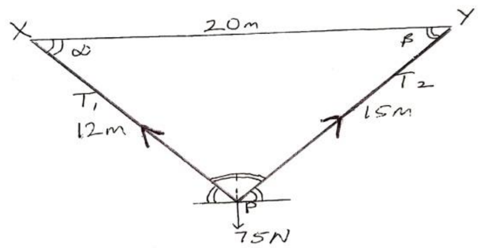
Candidates were required to illustrate forces in a diagram, use it to calculate angles and then to find the tensional forces in the string.

Most candidates could not draw the diagram and use it to solve the problem. Also, most candidates could not use the cosine and sine rules to solve the problem.

Candidates' performance was average.

Candidates were required to answer the question as follows:

14.(a)



$$cos\alpha = \frac{20^2 + 12^2 - 15^2}{(2)(20)(12)}$$
$$= \frac{400+144-225}{480} = \frac{319}{480} = 0.6646$$
$$\alpha = cos^{-1}0.6646 = 48.35^0$$
$$cos\beta = \frac{20^2 + 15^2 - 12^2}{(2)(20)(15)}$$
$$= \frac{400+225-144}{600} = \frac{481}{600} = 0.8017$$
$$\beta = cos^{-1}0.8017 = 36.71^0$$

(b) Using Lami's theorem

$$\frac{T_1}{sin126.71^0} = \frac{T_2}{sin138.35^0} = \frac{75}{sin94.94^0}$$
$$\frac{T_1}{sin126.71^0} = \frac{75}{sin94.94^0}$$
$$T_1 = \frac{75 \times sin126.71^0}{sin94.94^0} = \frac{75 \times 0.8017}{0.9963}$$
$$T_1 = 60.3495N$$
$$= 60.35N$$

(c)
$$\frac{T_2}{sin138.35^0} = \frac{75}{sin94.94^0}$$

$$T_2 = \frac{75 \times sin138.35^0}{sin94.94^0} = \frac{75 \times 0.6646}{0.9963}$$

$$T_2 = 50.0301N$$

$$= 50.03N$$

QUESTION 15

A particle *P* moves in a plane such that at time *t* seconds, its velocity, $v = (2t\mathbf{i} - t^3 j)\ m\ s^{-1}$.

(a) Find, when $t = 2$, the magnitude of the:

(i) velocity of *P*.

(ii) acceleration of *P*.

(b) Given that *P* is at the point with position vector $(3i + 2j)$ when $t = 1$, find the position vector of *P* when $t = 2$.

Candidates were required to use differentiation and integration of vectors (in *i*, *j*, *k*) to solve the problem. Most of the candidates scored good marks in the part (a). the part (b) was poorly done as a good number of the candidates could not integrate the velocity vector to obtain the position vector.

Candidates were required to answer the question as follows:

(a)(i) $v = 2ti - t^3 j$

When $t = 2,\ v = 4i - 8j$

$|v| = \sqrt{4^2 + (-8)^2} = \sqrt{80}$

$= 4\sqrt{5}ms^{-1} or\ 8.944ms^{-1}$

$a = 2 - 3t^2$

$a = 2i - \ 3t^2 j$

$= 2i - 12j$

(ii) $|a| = \sqrt{2^2 + (-12)^2} = \sqrt{148}$

$= 2\sqrt{37}ms^{-2}\ or\ 12.166ms^{-2}$

$S = \frac{2t^2}{2} i - \frac{t^4}{4} j$

$S = t^2 - \frac{t^4}{4} + k$

When t = 1, $(3i + 2j) = 1i - \frac{1}{4} j + k$

$3i - i + 2j + \frac{1}{4} j = k$

$\left(2i + \frac{9}{4} j\right) = k$

(b) $S = t^2 i - \frac{t^4}{4} j + 2i + \frac{9}{4} j$

$= (t^2 + 2)i + \left(\frac{9}{4} - \frac{t^4}{4}\right) j$

When t = 2, $S = 6i + \left(\frac{9}{4} - 4\right) j$

$S = 6i + - \frac{7}{4} j$

$S = 6i - \frac{7}{4} j$